Chapter III

Answers to the 1997 AP Calculus AB and Calculus BC Examinations

- Section I: Multiple Choice
  - Blank Answer Sheet
- Section II: Free Response
  - Student Preparation for the Exams
  - Free-Response Questions, Scoring Guidelines, and Sample Student Responses with Commentary
  - Section II, Calculus AB
  - Section II, Calculus BC

Section I: Multiple Choice

Listed below are the correct answers to the multiple-choice questions and the percentage of AP candidates who answered each question correctly. A copy of the blank answer sheet appears on the following pages for reference.

Section I Answer Key and Percent Answering Correctly

Calculus AB

Item No.	Correct Answer	Percent Correct	Item No.	Correct Answer	Percent Correct	Item No.	Correct Answer	Percent Correct
1	C	86%	15	B	59%	78	D	31%
2	A	57%	16	D	46%	79	C	40%
3	C	38%	17	A	22%	80	A	50%
4	D	78%	18	C	29%	81	A	22%
5	E	58%	19	D	38%	82	B	45%
6	C	69%	20	E	42%	83	C	60%
7	D	70%	21	E	34%	84	C	26%
8	C	68%	22	D	28%	85	C	52%
9	B	27%	23	A	30%	86	A	35%
10	E	56%	24	B	7%	87	B	48%
11	E	68%	25	A	49%	88	E	49%
12	B	51%	76	E	75%	89	B	47%
13	A	59%	77	D	61%	90	D	21%
14	C	34%						

Calculus BC

Item No.	Correct Answer	Percent Correct	Item No.	Correct Answer	Percent Correct	Item No.	Correct Answer	Percent Correct
1	C	85%	15	D	66%	78	A	71%
2	E	92%	16	B	78%	79	D	64%
3	A	61%	17	B	30%	80	B	70%
4	C	86%	18	C	62%	81	D	32%
5	C	88%	19	D	38%	82	B	56%
6	A	50%	20	E	52%	83	E	48%
7	C	78%	21	A	22%	84	C	65%
8	E	68%	22	C	65%	85	D	37%
9	A	34%	23	E	52%	86	A	67%
10	B	68%	24	D	28%	87	B	34%
11	C	45%	25	A	84%	88	C	29%
12	A	74%	76	D	56%	89	D	28%
13	B	55%	77	E	64%	90	B	56%
14	C	52%						

1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

   (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(a) $x(t) = \int v(t)\,dt = \int (3t^2 - 2t - 1)\,dt$

$= t^3 - t^2 - t + C$

$x(2) = 8 - 4 - 2 + C = 5\,; \quad C = 3$

$x(t) = t^3 - t^2 - t + 3$

3 {
- 2: $x(t) = t^3 - t^2 - t + C$
  - <-1> error(s) in $t^3 - t^2 - t$
  - <-1> no constant of integration
- 1: evaluates constant of integration

(b) avg. vel. $= \dfrac{x(3) - x(0)}{3 - 0}$

$= \dfrac{18 - 3}{3} = 5$

$3t^2 - 2t - 1 = 5$

$t = \dfrac{1 + \sqrt{19}}{3}$ or 1.786

3 {
- 1: average velocity $= 5$
- 1: sets $v(t)$ equal to student's average velocity
- 1: answer
  0/1 if not solving
  $3t^2 - 2t - 1 =$ an avg. velocity

(c) distance $= \int_0^3 |v(t)|\,dt$

$= \int_0^3 |3t^2 - 2t - 1|\,dt = 17$

or

$v(t) = 3t^2 - 2t - 1 = 0$

$t = -\dfrac{1}{3}\,, \quad t = 1$

$x(0) = 3$

$x(1) = 1 - 1 - 1 + 3 = 2$

$x(3) = 27 - 9 - 3 + 3 = 18$

distance $= (3 - 2) + (18 - 2) = 17$

3 {
- 1: limits of 0 and 3 on an integral of $v(t)$ or $|v(t)|$
  or
  substitutes $t = 0$ and $t = 3$ in $x(t)$
- 1: handles change in direction at student's turning point
- 1: answer

AB-2 1997

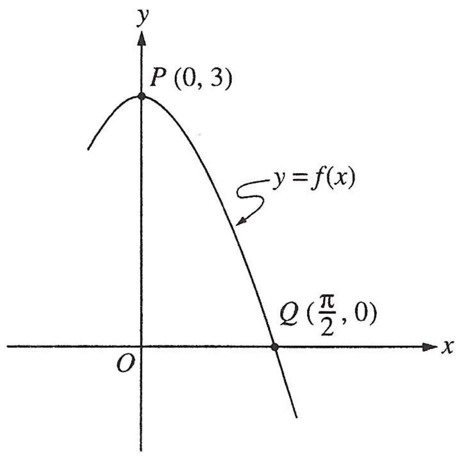
2. Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at point P and the x-axis at point Q.

   (a) Write an equation for the line passing through points P and Q.

   (b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.

   (c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.

   (d) Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.



(a) $\text{slope} = \dfrac{3-0}{0-\pi/2} = -\dfrac{6}{\pi}$

$y - 3 = -\dfrac{6}{\pi}(x - 0)$

$2\begin{cases} 1\text{: slope} \\ 1\text{: equation} \end{cases}$

(b) $f'(x) = -3\sin x$

$f'(\pi/2) = -3\sin(\pi/2) = -3$

$y - 0 = -3(x - \pi/2)$

$2\begin{cases} 1\text{: } f'(\pi/2) = -3 \\ 1\text{: equation using } f'(\pi/2) \text{ and } f(\pi/2) \end{cases}$

1/2 if equation only

(c) $f'(x) = -3\sin x = -\dfrac{6}{\pi}$

$\sin x = \dfrac{2}{\pi}$

$x = 0.690$

$2\begin{cases} 1\text{: equates derivative to slope} \\ 1\text{: solution in } [0, \pi/2] \end{cases}$

1/2 if solution only

(d) $V = \pi \displaystyle\int_0^{\pi/2} \left[(3\cos x)^2 - \left(-\frac{6}{\pi}x + 3\right)^2 \right] dx$

$3\begin{cases} 2\text{: integrand} \\ \quad 0/2 \text{ if not difference of 2 squares} \\ \quad 1/2 \text{ if incorrect but of form} \\ \quad (a\cos x)^2 - (bx + c)^2 \ ; \ a, b \neq 0 \\ \quad 1/2 \text{ if reversal} \\ 1\text{: constant and limits} \end{cases}$

3. Let f be the function given by $f(x) = \sqrt{x-3}$.

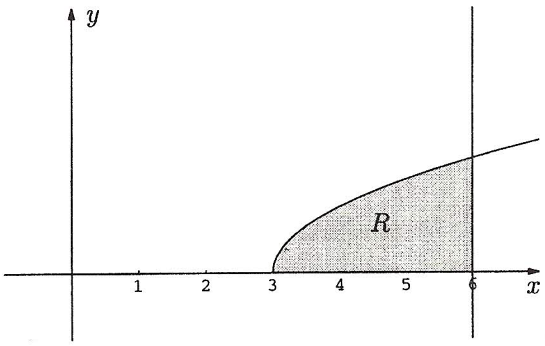
(a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.

(b) Find the area of the region R described in part (a).

(c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

(d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

(a)



2 { 1: graph of f, (domain is $x \geq 3$, goes through $(3, 0)$, is increasing, positive, and concave down)
1: correct region relative to graph of f }

(b) area $= \int_3^6 \sqrt{x-3}\, dx = \frac{2}{3}(x-3)^{3/2}\Big]_3^6$

$= 2\sqrt{3} = 3.464$

3 { 1: limits
1: integrand
1: answer
0/1 if second point is not earned }

(c) $A(w) = \int_3^w \sqrt{x-3}\, dx$

2 { 1: limits
1: integrand }

(d) $\frac{dA}{dw} = \sqrt{w-3}$

$\left.\frac{dA}{dw}\right|_{w=6} = \sqrt{3} = 1.732$

2 { 1: $\frac{dA}{dw}$
1: evaluation at 6 }

0/2 if $\frac{dA}{dw}$ is constant

4. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

   (a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

   (b) For what values of the constant p does f have 3 distinct roots?

   (c) Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.

(a) $f(x) = x^3 - 6x^2 + p$

$f'(x) = 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0 \ , \ x = 4$

$f'(x)$ changes sign from positive to negative at $x = 0$
$f'(x)$ changes sign from negative to positive at $x = 4$

or

$f''(x) = 6x - 12$, $f''(0) = -12$, $f''(4) = 12$

relative maximum at $x = 0$, $f(0) = p$

relative minimum at $x = 4$, $f(4) = p - 32$

4
- 1: finds $f'(x)$
- 1: solves $f'(x) = 0$
- 1: indicates the location of a maximum and a minimum, with analysis
  0/1 if explicitly chooses to work only with a specific value for p
- 1: finds the maximum and minimum values in terms of p

(b) $f(x)$ has three distinct real roots when $p > 0$ and $p - 32 < 0$,

so $0 < p < 32$

2
- 1: upper bound
- 1: lower bound

$<-1>$ including 0 or 32, or both

(c) $$\frac{1}{2-(-1)}\int_{-1}^{2}(x^3 - 6x^2 + p)\,dx = 1$$

$$\frac{1}{3}\left[\frac{1}{4}x^4 - 2x^3 + px\right]_{-1}^{2} = 1$$

$$\frac{1}{3}\left[\left(\frac{16}{4} - 16 + 2p\right) - \left(\frac{1}{4} + 2 - p\right)\right]$$

$$= \frac{1}{3}\left[3p - \frac{57}{4}\right] = 1$$

$$p = \frac{23}{4} = 5.75$$

3
- 2: average value
  - 1: integrand and limits
  - 1: appropriate constant for definite integral
- 1: sets average value equal to 1 and solves for p

  equation must be of the form

  $$k\int_{-1}^{2}(x^3 - 6x^2 + p)\,dx = 1$$

1/3 solution only

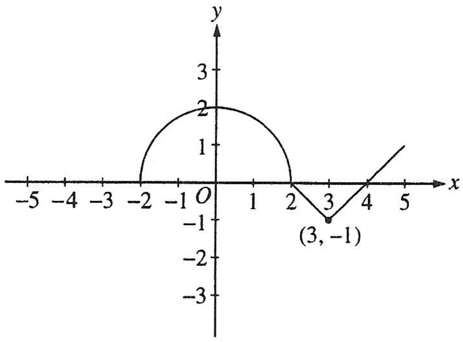
5. The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(3)$.

   (b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

   (c) Write an equation for the line tangent to the graph of g at $x = 3$.

   (d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.



(a) $g(3) = \int_0^3 f(t)\,dt$ $= \frac{1}{4}\cdot\pi\cdot 2^2 - \frac{1}{2} = \pi - \frac{1}{2}$	**2** { 2: answer <−1> each incorrect area <−1> error in summing
(b) $g(x)$ has relative maximum at $x = 2$ because $g'(x) = f(x)$ changes from positive to negative at $x = 2$	**3** { 1: relative maximum at $x = 2$ only 1: $g'(x) = f(x)$ or interprets $g(x)$ as area accumulator 1: justification (ignore discussion at $x = 5$)
(c) $g(3) = \pi - \frac{1}{2}$ $g'(3) = f(3) = -1$ $y - \left(\pi - \frac{1}{2}\right) = -1(x - 3)$	**2** { 1: $g'(3) = -1$ 1: equation using $g(3)$ and $g'(3)$
(d) graph of g has points of inflection with x-coordinates $x = 0$ and $x = 3$ because $g''(x) = f'(x)$ changes from positive to negative at $x = 0$ and from negative to positive at $x = 3$ or because $g'(x) = f(x)$ changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = 3$	**2** { 1: points of inflection with x-coordinates 0 and 3 only 1: justification (ignore discussion at $x = 2$) 1/2 if $x = 0, 3$ selected as candidates and $x = 3$ discarded because $g''(3)$ does not exist 1/2 if $x = 0, 2, 3$ selected as candidates and $x = 2$ and $x = 3$ discarded because $g''(2)$ and $g''(3)$ do not exist

6. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

   (a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

   (b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

   (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

(a) $\dfrac{dv}{dt} = -2v - 32 = -2(v + 16)$

$\dfrac{dv}{v + 16} = -2\,dt$

$\ln|v + 16| = -2t + A$

$|v + 16| = e^{-2t+A} = e^{A}e^{-2t}$

$v + 16 = Ce^{-2t}$

$-50 + 16 = Ce^{0}\ ; \quad C = -34$

$v = -34e^{-2t} - 16$

6 $\begin{cases} \text{1: separates variables} \\ \text{1: antiderivative of } dv \text{ side} \\ \quad \text{0/1 if not } \displaystyle\int \frac{dv}{av+b},\ a \neq 0 \\ \text{1: antiderivative of } dt \text{ side} \\ \text{1: constant of integration} \\ \text{1: uses initial condition } v(0) = -50 \\ \text{1: solves for } v(t) \\ \quad \text{0/1 if not solving } \dfrac{dv}{av+b} = k\,dt \\ \quad \text{where } a,\ b,\ k \text{ nonzero} \\ \quad \text{0/1 if no constant of integration} \end{cases}$

0/6 if variables not separated

(b) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty} \left(-34e^{-2t} - 16\right) = -16$

1: limit value
must be exponential $v(t)$ with finite limit

(c) $v(t) = -34e^{-2t} - 16 = -20$

$e^{-2t} = \dfrac{2}{17}\ ; \quad t = -\dfrac{1}{2}\ln\left(\dfrac{2}{17}\right) = 1.070$

2 $\begin{cases} \text{1: sets } v(t) = -20 \\ \text{1: solution} \\ \quad \text{must be exponential } v(t) \end{cases}$